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Some Fixed Point Theorems for G-Nonexpansive Mappings on Ultrametric Spaces and Non-Archimedean Normed Spaces with a Graph

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ABSTRACT: A very interesting approach in the theory of fixed point is some general structures was recently given by Jachymski by using the context of metric spaces endowed with a graph. The purpose of this article is to present some new fixed point results for G-nonexpansive mappings defined on an ultrametric space and non-Archimedean normed space which are endowed with a graph. In particular, we investigate the relationship between weak connectivity graph and the existence of fixed point for these mappings.

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1. Introduction and Preliminaries

In 2004, Ran and Reurings [16] started a new research direction in fixed point theory by proving the following Banach-Caccioppoli type principle in ordered metric spaces. Later, in 2005, Nieto and Rodriguez-López [9] proved a modified variant of Ran and Reurings's result by removing the continuity of selfmaps. Notice that the case of decreasing operators is treated by Nieto and Rodríguez-López [10], where some interesting applications to ordinary differential equations with periodic boundary conditions are also given. Also, Nieto, Pouso and Rodríguez-López improved in a recent paper [8] some results on the same topic given by Petrusel and Rus [14] by working in the setting of abstract L-spaces in the sense of Fréchet [8]. Very recently, Agarwal, El-Gebeily and OŔegan extended in [1] the result of Ran and Reurings [16] for the case of generalized ϕ-contractions, while O'Regan and Petrusel [11] proved some new

fixed point results for ϕ-contractions on ordered metric spaces with applications to integral equations. The case of weakly contractive mappings in ordered metric spaces is treated by Harjani and Sadarangani in [5]. A very interesting approach was given by Jachymski [6] and by Gwóźdź-Łukawska and Jachymski [4], where the authors studied the case of self-operators on metric spaces endowed with a graph.
In this paper, we first recall some basic notions in ultrametric spaces and non-Archimedean normed spaces, and motivated by the works of Petalas and Vidalis [12], Kirk and Shazad [7] and Jachymski [6], introduce two new conditions for non-expansive mappings on complete ultrametric spaces (non-Archimedean spaces) and, using these conditions, obtain some fixed point theorems.
The founding father of non-Archimedean functional analysis was Monna, who wrote a series of paper in 1943. A milestone was reached in 1978 at the publication of van Rooij's book [17], the most extensive treatment on non-Archimedean Banach spaces existing in the literature. For more details the reader is referred to [3, 13, 17]. The idea is reasonable to try and generalize ordinary functional analysis by replacing $\mathbb{R}$ and $\mathbb{C}$ by other topological field. This ought to give a new insight in analysis by showing what properties of the scalar field are crucial for certain classical theorems. For this topological field Monna choose a field $\mathbb{K}$, provided with real valued absolute value function $|\cdot|$ such that $\mathbb{K}$ is complete relative to the metric induced by $|\cdot|$. Adding the condition that, as a topological field, $\mathbb{K}$ is neither $\mathbb{R}$ nor $\mathbb{C}$, Monna proved the so-called strong triangle inequality

$$|x+y| \leq \max\{|x|, |y|\} \qquad (x, y \in \mathbb{K}).$$

This formula is essential to theorems in non-Archimedean functional analysis. Among other things it implies that $\mathbb{K}$ is totally disconnected and cannot be made into a totally ordered field [17].
Van Rooij [17] introduced the concept of ultrametric space as follows:
Let (X, d) be a metric space. (X, d) is called an ultrametric space if the metric d satisfies the strong triangle inequality, i.e., for all $x, y, z \in X$:

$$d(x, y) \leq \max\{d(x, z), d(y, z)\},$$

in this case d is said to be ultrametric. We denote by $B(x, r)$, the closed ball

$$B(x, r) = \{y \in X : d(x, y) \leq r\},$$

where $x \in X$ and we let $r \geq 0$, $B(x, 0) = \{x\}$. A known characteristic property of ultrametric spaces is the following:

If $x, y \in X$, $0 \leq r \leq s$ and $B(x, r) \cap B(y, s) \neq \emptyset$, then $B(x, r) \subset B(y, s)$.

An ultrametric space (X, d) is said to be spherically complete if every shrinking collection of balls in X has a nonempty intersection. A non-Archimedean valued field is a field $\mathbb{K}$ equipped with a function (valuation) $|\cdot|$ from $\mathbb{K}$ into $[0, \infty)$ such that $|x| = 0$ if and only if $x = 0$, $|x+y| \leq \max\{|x|, |y|\}$ and $|xy| = |x||y|$ for all $x, y \in \mathbb{K}$. Clearly, $|1| = |-1| = 1$ and $|n.1_{\mathbb{K}}| \leq 1$ for all $n \in \mathbb{N}$ [17].

An example of a non-Archimedean valuation is the mapping $|\cdot|$ taking each point of an arbitrary field but 0 into 1 and $|0| = 0$. This valuation is called trivial. The set $\{|x| : x \in \mathbb{K}, x \neq 0\}$ is a subgroup of the multiplicative group $(0, +\infty)$ and it is called the value group of the valuation. The valuation is called trivial, discrete, or dense accordingly as its value group is $\{1\}$, a discrete subset of $(0, +\infty)$, or a dense subset of $(0, \infty)$, respectively [17]. A norm on a vector space X over a non-Archimedean valued field $\mathbb{K}$ is a map $\|\cdot\|$ from X into $[0, \infty)$ with the following properties:

1) $\|x\| \neq 0$ if $x \in X \setminus \{0\}$;
2) $\|x + y\| \leq \max\{\|x\|, \|y\|\} \qquad (x, y \in X)$;
3) $\|\alpha x\| = |\alpha|\|x\| \qquad (\alpha \in \mathbb{K}, x \in X)$.

In1993, Petalas and Vidalis in [12] presented a generalization of a well-known fixed point theorem for the class of spherically complete non-Archimedean normed spaces, and in 2000 Priess-Crampe and Ribenboim in [15] obtained similar results in ultrametric space, but the proofs of these theorems weren't constructive. In 2012 Kirk and Shahzad in [7] gave more constructive proofs of these theorems and strengthened the conclusions as follow:

Theorem 1.1 ([7]). *Suppose that (X, d) is a spherically complete ultrametric space and $T : X \longrightarrow X$ is a nonexpansive mapping (i.e., $d(Tx, Ty) \leq d(x, y)$ for every x and y in X). Then every closed ball of the form*

$$B(x, d(x, Tx)) \qquad (x \in X),$$

contains either a fixed point of T or a minimal T-invariant closed ball.
Where a ball $B(x, r)$ is called T-invariant if $T(B(x, r)) \subset B(x, r)$ and is called minimal T-invariant if $B(x, r)$ is T-invariant and $d(u, Tu) = r$ for all $u \in B(x, r)$.

2. Main Results

Let $G = (V(G), E(G))$ be a directed graph. By $\tilde{G}$ we denote the undirected graph obtained from G by ignoring the direction of edges. If x and y are two vertices in a graph G, then a path in G from x to y of length n is a sequence $(x_i)_{i=0}^{n}$ of $n + 1$ vertices such that $x_0 = x$, $x_n = y$ and $(x_{i-1}, x_i) \in E(G)$ for $i = 1, \ldots, n$, we always suppose that paths are of the shortest length. A graph G is called connected if there is a path between any two vertices and is called weakly connected if $\tilde{G}$ is connected. Subsequently, in this paper X is a complete ultrametric space or non-Archimedean normed space with ultrametric d, Δ is the diagonal of the Cartesian product $X \times X$ and G is a directed graph such that the set $V(G)$ of its vertices coincides with X, the set $E(G)$ of its edges contains Δ and G has no parallel edges. Moreover, we may treat G as a weighted graph by assigning to each edge the distance between its vertices. We first give our two results with constructive proofs. In fact, we generalize Kirk and Shahzad's result on nonexpansive mappings on ultrametric spaces and non-Archimedean normed spaces endowed with a graph.

Definition 2.1. Let (X, d) be a metric space endowed with a graph G. We say that a mapping $T : X \longrightarrow X$ is G-nonexpansive if

1) T preserves the edges of G, i.e., $(x, y) \in E(G)$ implies $(Tx, Ty) \in E(G)$ for all $x, y \in X$; and

2) $d(Tx, Ty) \leq d(x, y)$ for all $x, y \in X$ with $(x, y) \in E(G)$.

Definition 2.2. Suppose that (X, d) is an ultrametric space endowed with a graph G and $T : X \longrightarrow X$ a mapping. We would say that a ball $B(x, r)$ is graphically T-invariant if for any $u \in B(x, r)$ that there exists a path between u and x in $\tilde{G}$ with vertices in $B(x, r)$, we have

$$Tu \in B(x, r).$$

Also, a ball $B(x, r)$ is graphically minimal T-invariant if $Tu \in B(x, r)$ and $d(u, Tu) = r$ for any $u \in B(x, r)$ that there exists a path between u and x in $\tilde{G}$ with vertices in $B(x, r)$.

Theorem 2.3. *Let (X, d) be an ultrametric space endowed with a graph G, and G-nonexpansive mapping $T : X \longrightarrow X$ satisfies the following conditions:*

(A) *There exists an $x_0 \in X$ such that $d(x_0, Tx_0) < 1$;*

(B) *If $d(x, Tx) < 1$, then there exists a path in $\tilde{G}$ between x and Tx with vertices in $B(x, d(x, Tx))$;*

(C) *If $\{B(x_n, d(x_n, Tx_n))\}$ is a nonincreasing sequence of closed balls in X and for each $n \geq 1$, there exists a path in $\tilde{G}$ between x_n and x_{n+1} with vertices in $B(x_n, d(x_n, Tx_n)$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ and there exists $z \in \bigcap_{k=1}^{\infty} B(x_{n_k}, r_{n_k})$ such that for each $k \geq 1$, there exists a path in $\tilde{G}$ between x_{n_k} and z with vertices in $B(x_{n_k}, d(x_{n_k}, Tx_{n_k}))$.*

Then for each $x \in X$ with $d(x, Tx) < 1$, the closed ball $B(x, d(x, Tx))$ contains a fixed point of T or a graphically minimal T-invariant ball.

Proof. Let $x \in X$ with $d(x, Tx) < 1$, $r = d(x, Tx)$ and $u \in B(x, r)$ be such that there exists a path $(x_0 = x, x_1, x_2, \ldots, x_n = u)$ in $\tilde{G}$ with vertices in $B(x, r)$ between u and x. If $u = x$ then $Tu = Tx \in B(x, r)$, and if not, since $x \neq u$ and the path $(x_0 = x, x_1, \ldots, x_n = u)$ has the shortest lengh, we infer that for each i, $x_{i-1} \neq x_i$. Thus

$$\begin{aligned}
d(u, Tu) &\leq \max\{d(u, x), d(x, Tx), d(Tx, Tu)\} \\
&\leq \max\{d(u, x), d(x, Tx), d(Tx, Tx_1), d(Tx_1, Tx_2), ..., d(Tx_{n-1}, Tu)\} \\
&\leq \max\{d(u, x), d(x, Tx), d(x, x_1), d(x_1, x_2), ..., d(x_{n-1}, u)\} \\
&= d(x, Tx).
\end{aligned}$$

On the other hand, since $B(x, d(x, Tx)) \cap B(u, d(u, Tu)) \neq \emptyset$, we have $B(u, d(u, Tu)) \subset B(x, d(x, Tx))$, so $Tu \in B(x, d(x, Tx))$. This means that

$B(x, d(x, Tx))$ is graphically T-invariant. Now, fix $x_0 \in X$ with $d(x_0, Tx_0) < 1$ and let $x_1 = x_0$, $r_1 = d(x_1, Tx_1)$,

$$E_1 = \{x \in B(x_1, r_1) \mid \text{ there is a path in } \tilde{G} \text{ between } x \text{ and } x_1 \text{ with vertices in } B(x_1, r_1)\},$$

and

$$\mu_1 = \inf\{d(x, Tx) : x \in E_1\}.$$

Suppose $\{\epsilon_n\}$ is a sequence of positive numbers such that $\lim_{n\longrightarrow\infty} \epsilon_n = 0$. If $\mu_1 = r_1$, then the proof is completed because in this case either $r_1 = \mu_1 = 0$ therefore x_1 is a fixed point of T in $B(x_1, r_1)$ or $B(x_1, r_1)$ is graphically minimal T-invariant. Now, let $\mu_1 < r_1$. Choose $x_2 \in B(x_1, r_1)$ such that there exists a path in $\tilde{G}$ between x_1 and x_2, and

$$r_2 = d(x_2, Tx_2) < \min\{r_1, \mu_1 + \epsilon_1\}.$$

Suppose that by induction x_n is obtained. Put

$$E_n = \{x \in B(x_n, r_n) \mid \text{ there is a path in } \tilde{G} \text{ between } x \text{ and } x_n \text{ with vertices in } B(x_n, r_n)\},$$

and

$$\mu_n = \inf\{d(x, Tx) : x \in E_n\}.$$

If $r_n = 0$ or $\mu_n = r_n$, using the similar argument as for n=1, the proof is complete. Otherwise, choose $x_{n+1} \in B(x_n, r_n)$ such that

$$r_{n+1} = d(x_{n+1}, Tx_{n+1}) < \min\{r_n, \mu_n + \epsilon_n\}.$$

If this process ends after a finite number of steps, then we are done. Otherwise, proceeding in the same manner, we obtain a nonincreasing sequence $\{B(x_n, d(x_n, Tx_n))\}$ of nontrivial closed balls. Since $\{r_n\}$ is nonincreasing, $r := \lim_{n\longrightarrow\infty} r_n$ exists. Also, $\{\mu_n\}$ is nondecreasing and bounded above, thus, $\mu := \lim_{n\longrightarrow\infty} \mu_n$ also exists. Hence by (C), there exists a subsequence $\{x_{n_k}\}_{k=1}^{\infty}$ of $\{x_n\}_{n=1}^{\infty}$ and $z \in \cap_{n=1}^{\infty} B\big(x_{n_k}, r_{n_k}\big)$ such that for each $k \in \mathbb{N}$ there exists a path in $\tilde{G}$ between x_{n_k} and z with vertices in $B(x_{n_k}, d(x_{n_k}, Tx_{n_k}))$. Since $B(x_{n_k}, r_{n_k})$ is graphically T-invariant for all $k \geq 1$, it follows that $Tz \in B(x_{n_k}, r_{n_k})$, for all $k \geq 1$. Therefore,

$$\begin{aligned} \mu_{n_k+1} &\leq d(z, Tz) \\ &\leq \max\{d(x_{n_k}, z), d(x_{n_k}, Tz)\} \\ &\leq r_{n_k}, \end{aligned}$$

for all $k \geq 1$. Thus,

$$\begin{aligned} \mu_{n_k+1} &\leq d(z, Tz) \\ &\leq r \\ &\leq r_{n_k+1} \\ &\leq \mu_{n_k} + \epsilon_{n_k}, \end{aligned}$$

for all $k \geq 1$. Letting $k \longrightarrow \infty$, we obtain $d(z, Tz) = r = \mu$. On the other hand, if $x \in B(z, d(z, Tz))$, then $d(x, z) \leq d(z, Tz) \leq r_{n_k}$ for each $k \in \mathbb{N}$. Therefore,

$$d(x, x_{n_k}) \leq \max\{d(x, z), d(x_{n_k}, z)\} \leq r_{n_k},$$

for all $k \geq 1$. Hence, $x \in B(x_{n_k}, r_{n_k})$ for all $k \geq 1$. Now, let $x \in B(z, d(z, Tz))$ and there exists a path between x and z. Thus, there exists a path in $B(z, d(z, Tz))$ between x_{n_k} and x for all $k \geq 1$. Hence $\mu_{n_k} \leq d(x, Tx)$ for all $k \geq 1$. Therefore, for each $k \in \mathbb{N}$, $\mu_{n_k} \leq r_{n_k}$. Hence

$$\inf\{d(x, Tx) : x \in B(z, d(z, Tz))\} = d(z, Tz) = r = \mu.$$

If $r = 0$, then z is a fixed point of T in $B(x, d(x, Tx))$, if not, then the closed ball $B(z, d(z, Tz))$ is graphically minimal T-invariant. Therefore the proof is completed. □

Corollary 2.4. Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a metric d in X such that (X, d) is a spherically complete ultrametric space, and $G = (V(G), E(G))$ is a directed graph with $V(G) = X$ and $E(G) = \{(x, y) \in X \times X : x \preceq y\}$. Suppose also that $T : X \longrightarrow X$ is a G-nonexpansive mapping such that (A), (B) and (C) in Theorem 2.3 hold. Then for every $x \in X$ with $d(x, Tx) < 1$, the closed ball $B\big(x, d(x, Tx)\big)$ contains a fixed point of T or a graphically minimal T-invariant ball.

Remark 2.5. Theorem 2.3 remains valid if the ultrametric space (X, d) is replaced by a spherically complete non-Archimedean normed space $(X, \|\cdot\|)$. Also, note that Corollary 2.4 is valid if the ultrametric space (X, d) is replaced with a non-Archimedean normed space $(X, \|\cdot\|)$.

In the previous theorem, we obtained some results on a closed balls $B(x, d(x, Tx))$ with $d(x, Tx) < 1$. In the following Theorem we obtain these results on every weakly connected ball of the form $B(x, d(x, Tx))$ by adding weak connectivity.

Theorem 2.6. *Let (X, d) be a spherically complete ultrametric space endowed with graph G and let $T : X \longrightarrow X$ be a G-nonexpansive mapping. Suppose also that for each $x \in X$ the ball $B(x, d(x, Tx))$ is weakly connected. Then for each $z \in X$ the closed ball $B(z, d(z, Tz))$ contains either a fixed point of T or a minimal T-invariant ball.*

Proof. Let $x \in X$, and $\Gamma = \{B(y, d(y, Ty)) : y \in B(x, d(x, Tx))\}$. Γ can be partially ordered by set inclusion. Then, using Zorn's Lemma, Γ has a minimal element, say $B(z, d(z, Tz))$. If $B(z, d(z, Tz))$ is singleton, then z is a fixed point of T. If not, we show $B(z, d(z, Tz))$ is minimal T-invariant. It is easy to see that the ball $B(z, d(z, Tz))$ is T-invariant. On the other hand, for each $u \in B(z, d(z, Tz))$, we have

$$\begin{aligned} d(u, Tu) &\leq \max\{d(u, z), d(Tu, z)\} \\ &\leq d(z, Tz), \end{aligned}$$

so $d(u,Tu) = d(z,Tz)$, because if $d(u,Tu) \neq d(z,Tz)$, then $B(u,d(u,Tu)) \subset B(z,d(z,Tz))$ and $B(u,d(u,Tu)) \neq B(z,d(z,Tz))$. This contradicts the minimality of $B(z,d(z,Tz))$. Therefore, $B(z,d(z,Tz))$ is minimal T-invariant. $\square$

Remark 2.7. Theorem 2.6 remains valid if the ultrametric space (X,d) is replaced by a spherically complete non-Archimedean normed space $(X,\|\cdot\|)$.

3. Examples

In this section, we will give some examples to support our theorems. We also compare the hypotheses of Theorems 2.3 and 2.6 in Examples 2 and 3. In the first example, we present a spherically complete ultrametric space endowed with a weakly connected graph to support Theorem 2.6.

Example1. Let X be the space c_0 over a non-Archimedean valued field $\mathbb{K}$ with the valuation of $\mathbb{K}$ discrete and choose $\pi \in \mathbb{K}$ with $0 < |\pi| < 1$. Define graph $G = (V(G), E(G))$ by $V(G) = X$ and

$$E(G) = \{(x,y) \in X \times X : \text{ either } x = y \text{ or there exists exactly one } i \in \mathbb{N} \text{ such that } x_i = y_i\}.$$

Let $B(x,r)$ be an arbitrary closed ball in X and let $y,z \in B(x,r)$. If $y = z$, then (z,y) is a path in $\tilde{G}$ from z to y. Otherwise, we have two cases: Either there exists an $i \in \mathbb{N}$ such that $y_i = z_i$ or not.

case 1. Let there exist $i \in \mathbb{N}$ such that $y_i = z_i$. For each $j \neq i$ we define w_j in the following way:

$$w_j = \begin{cases} y_j + z_j & y_j \neq 0, z_j \neq 0, \\ \pi(y_j + z_j) & \text{either } y_j = 0, \text{or } z_j = 0, \\ \pi^{n_j} & \text{there exists } n_j \text{ such that } |n_j| < 1,\ n_{j+1} > n_j. \end{cases}$$

Now, put

$$w = (w_1, w_2, ...w_{i-1}, z_i, w_{i+1}, ...).$$

The process of creating of $\{w_k\}$ shows that for each $j \neq i$, $w_j \neq z_j, y_j$ and $|w_j| < r$. Since $\{n_j\}$ is an increasing sequence $\lim_{j\to\infty} |\pi^{n_j}| = 0$. On the other hand, $\lim_{j\to\infty} |z_j + y_j| = 0$ and $\lim_{j\to\infty} |\pi(z_j + y_j)| = 0$. Thus $w \in c_0$, and also for each j, $|w_j| < r$, so $w \in B(0,r)$.

case 2. Let for each $i \in \mathbb{N}$, $z_i \neq y_i$. Put $w = (z_1, y_2, w_3, w_4, ...)$, where w_j is defined as case 3.

Then for any case (z,w,y) is a path between z and y with vertices in $B(x,r)$. Therefore, $B(x,r)$ is weakly connected. It is well known when $\mathbb{K}$ is a non-Archimedean valued field with the valuation of $\mathbb{K}$ discrete, c_0 is spherically complete. Therefore, all conditions of Theorem 2.6 hold. On the other hand, if there exists $x_0 \in X$ such that $d(x_0, Tx_0) < 1$ for G-nonexpansive mapping $T : c_0 \longrightarrow c_0$, the hypothesis (B) of the Theorem 2.3 hold.

In the following example, we show conditions of Theorem 2.3 are independent of conditions of Theorems 2.6.

Example2. Let X be the space c_0 over a non-Archimedean valued field $\mathbb{K}$ with the valuation of field $\mathbb{K}$ discrete. Suppose $w \in B(0,1)$ has exactly one zero coordinate. Define graph G', with $V(G') = X$, and

$$E(G') = \{(x,y) \in X \times X : x = y \\ \text{or } (x,y) \in E(G), (x,w) \in E(G) \text{ and } (y,w) \in E(G)\}.$$

It is obvious that G' isn't weakly connected. Therefore conditions of Theorem 2.6 do not hold. Now, define $T : X \longrightarrow X$ by

$$T(x) = \begin{cases} (x_1, x_2, x_3, ...), & (x,w) \in E(G), \\ (1 + x_1, 2x_2, 2x_3, ...), & \text{otherwise}. \end{cases}$$

T is a G'-nonexpansive mapping. It can be readily seen that conditions of Theorem 2.3 hold.

The following example shows that the hypotheses of Theorem 2.6 are independent of the hypotheses of Theorem 2.3.

Example3. Let X be the space c_0 over a non-Archimedean valued field $\mathbb{K}$ with the valuation of $\mathbb{K}$ discrete. We consider X with the graph G defined in Example 3. Let $e \in \mathbb{K}$ with $|e| > 1$. As we have shown in Example 3, for every G-nonexpansive mapping T the hypotheses of Theorem 2.6 hold. Define $T : X \longrightarrow X$ by

$$T(x_1, x_2, \ldots) = (e, x_1, x_2, \ldots),$$

for each $x \in X$. We have

$$d(x, Tx) = \sup\{|x_1 - e|, |x_2 - x_1|, |x_3 - x_2|, ...\},$$

so $|x_1 - e| \leq d(x, Tx)$. Since $|x_1 - e| = \max\{|x_1|, |e|\}$ and $|e| \geq 1$, we infer $d(x, Tx) \geq 1$ for all $x \in X$. Hence for each $x \in X$, $d(x, Tx) \geq 1$ and the hypotheses of Theorem 2.3 do not hold.

Remark 3.1. It would be interesting to compare our results with results obtained by Alfuraidan in [2] that investigated the fixed point theorems for nonexpansive mappings in Archimedean Banach normed spaces.

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